City Digits: Local Lotto

Curriculum Overview

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1. Introduction to the Lottery: What do we know, what don’t we know?

The unit begins with a collective brainstorm of students’ knowledge about the lottery and having students share their ideas about what they might do if they won. Students then visualize what they already know about how the lottery works by creating images that represent their understanding of the lottery system. This helps students identify gaps in their shared knowledge and develop questions about how the lottery works as a system.
2. Chances

2.1 Exploring Games of Chance

Why do we win more in some games of chance than in others? In this lesson, students play a series of games of chance: 1) straight-bet roulette, 2) a “color pick” game involving choosing three colors from a set of 5 colors, and 3) Sweet Millions, a local lottery game. Students compare win and loss data from aggregated classroom trials of each game. They formulate ideas about the probability of winning each game, paying specific attention to the role of parameters.

COLOR PICK GAME

<table>
<thead>
<tr>
<th>Round</th>
<th>Your color picks</th>
<th>Winning colors</th>
<th># of wins in the class</th>
<th># of losses in the class</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>YOB</td>
<td>2</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>YRB</td>
<td>1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>YOB</td>
<td>2</td>
<td>14</td>
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<td>4</td>
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<td>5</td>
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<td>6</td>
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<td>7</td>
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</tbody>
</table>
2.2 Counting Outcomes

Is it necessary to play a game many times to figure out the likelihood of winning? If not, how can we measure this likelihood? In this lesson, students learn that they can find the probability of winning a game by finding the ratio all the possible ways to win to all the possible outcomes for the game. Students work in groups to create a collective probability tree representing all possible outcomes in the “color pick” game. Using features of the tree model, students identify patterns to come up with the probability of winning the game.

Additional Resources:
2.2 Counting outcomes lesson plan: see pages 9-12
2.2 Counting outcomes powerpoint slides: download slides at citydigits.org/#curriculum
2.3 Winning the Jackpot

What is the probability of winning the jackpot lotteries in New York? And how can we calculate them without drawing a tree diagram? In many cases, the number of choices to consider for calculating probability is too large to be able to draw as a tree diagram. In this lesson, students reflect upon the “color pick” probability tree from the previous lesson to draw out the mathematical principles of combinatorics and probability including the multiplication principle, permutations, and combinations. Students then apply these principles to find the probability of winning various local lottery game jackpots.

**Additional Resources:**
2.3 Summary sheet handout: see page 13
2.3 Summary sheet student work: see page 14

2.4 What is 4 million?

Students compare the probability of winning the various jackpot lotteries in New York and consider what the probabilities mean. Students scale very large numbers to familiar, smaller numbers.

**Additional Resources:**
2.4 What is 4 million powerpoint slides: download slides at citydigits.org/#curriculum

2.5 “Hey, Now You Know!”

The New York Lottery’s slogan is “Hey, You Never Know.” In this lesson, students use mathematics to respond to this slogan by revealing and explaining the unsaid facts about the lottery that are not given on their website. Students create posters that illuminate the mathematics of various local lottery games.
Did You Know...?

The probability of winning Powerball is sooooo misleading...

Betcha didn't know that chances of winning the million dollar prize is 1 out of 175,223,510!

That's like buying 438,059 bags of Skittles... and finding a single Skittle with a W on it.

- One 16 oz bag of skittles holds approximately 400 Skittles.

\[
\frac{175,223,510}{400} = 438,059.
\]

\(\Rightarrow\) Doesn't she look happy??!

BUT what she doesn't know is in order for her to win the JACKPOT...

That means... Gloria won't be so happy... when she's part of the 3185882 winners...!

NOW YOU KNOW...

- Your most likely key to get struck by lightning...

- Overall chances of winning: 1 out of 8!

- You need to lose 7 times to win 1 time.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>65,630.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td>169.94</td>
<td>575,757</td>
</tr>
<tr>
<td>Third</td>
<td>5,410.26</td>
<td>575,757</td>
</tr>
<tr>
<td>Fourth</td>
<td>59,850</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>65,630.25</td>
<td>575,757</td>
</tr>
</tbody>
</table>

One out of every 8.71 you would win.

But your most likely to win a free quick pick for free!
3. Lottery in our City

3.1 Interviews
In this session, students conduct interviews with people and with shopkeepers in the school’s neighborhood. They gather data on who plays or does not play the lottery and why as well as which stores sell or do not sell lottery tickets. The interviews are recorded with photographs and audio. Each interview is tagged to its location and becomes part of an online map that displays all of the interviews that were conducted.

Additional Resources:
View interviews at: citydigits.mit.edu

3.2 Analyzing Interviews
What kind of information did we get from our interviews? In this lesson, students seek out examples of different perspectives on the lottery from their interviewees and share interview results with the rest of the class.

Additional Resources:
3.2 Interview scavenger hunt cards: page 15

3.3 Local Lottery Spendings as a Percentage of Income
Students become familiar with maps and mapping conventions by exploring the Lotta Facts digital choropleth maps of New York City showing median family income by neighborhood. Students embody the concept of median household income by playing the role of households with differing incomes, lining up, and identifying the median family income. Students are introduced to ratio tables as a way to compare lottery spending to household income across neighborhoods with varying incomes. Students use the percentage of income map on the Lotta Facts tool to compare lottery spending across neighborhoods relative to median family household income.

Additional Resources:
View maps at: citydigits.mit.edu
3.3 Who spends more student work: page 16
3.4 Aggregated Neighborhood Losses (or State Profits?)

Students analyze maps with data on percentage of income spent on the lottery and median household incomes to understand financial losses at the neighborhood level. Students locate map data containing median household income per day and scale household income to weekly and monthly values for a specific neighborhood. Students use map statistics on the percentage of income spent on lottery in a day for their neighborhood to calculate money spent on the lottery in a week, and a month. Students are presented with actual distribution of lottery profits. Students discuss how data and information from today’s session could support an argument that the lottery is a regressive tax.

Additional Resources:
View maps at: citydigits.mit.edu

4. Opinions with Justifications - Student Lottery “Tours”

In small groups, students prepare a digital story board presentation to demonstrate their understanding of the lottery. The story board is an opinion story with evidence from mathematical analyses (Hey, Now You Know!), interviews, and map analyses. Students formulate their opinion statements and plan their story boards with sketches and notes on worksheets. Next, students finalize their storyboards and create their Lottery Tour storyboards on tablets/laptops. The tool archives student presentations for sharing with classmates and others.

Additional Resources:
View tours at: citydigits.mit.edu
Additional Resources
2.2 Counting Outcomes

Overview of the lesson plan:
Is it necessary to play a game many times to figure out the likelihood of winning? If not, how can we measure this likelihood? In this lesson, students learn that they can find the probability of winning a game by finding the ratio all the possible ways to win to all the possible outcomes for the game.

Daily goals:
Students will
1. Learn to calculate how many “tickets” there are by using tree diagrams to count.
2. Identify winning “tickets” on the tree diagram, understanding that there are multiple possible combinations to make a winning ticket.
3. Represent the probability as a fraction based on the tree diagram.

Summary of activities:
- **Introduction to counting outcomes and probability**: Teacher uses the single-bet roulette example in Chances Part I to introduce the concept of counting outcomes and writing probability as a fraction.
- **Creating a probability tree**: Students create a probability tree to represent all possible outcomes in the Color Pick game. Students identify and count winning “tickets” on the tree to come up with the probability of winning the game.

Mathematical ideas
This lesson introduces students to the idea of finding probability by counting outcomes. Students explore the meaning of what an outcome is and learn to create a tree diagram to count the outcomes in order to find the probability.

Making a model: In this lesson, students learn about the importance of having a systematic way to count all possible outcomes in a combination or permutation situation. The tree diagram is one specific tool that students learn to use in this lesson. The tree diagram provides a way for students to visually organize the possible outcomes so that there is less chance of missing an outcome. Some students may already have their own way of counting systematically; if so, help them make connections with the tree diagram so that they will have multiple tools for counting outcomes in the future.

Materials:
- PowerPoint for Chances 2
- Color branches and leaves in labeled bags
- Chart paper, 1 for each group
- Glue stick, 1 for each group
- Scissors, 1 for each group
- Tape, 1 for each group
Lesson Plan Outline

**12:00pm | Introduction to probability (10 min)**

- Do now (Slide 2)

  **Do Now**
  
  Silently write your answer to the following question in your notebook:
  
  Remember yesterday’s Color Combination game. Why did we have more luck winning this game than roulette or the Sweet Million lottery?

  The Do Now question serves as a warm up to remind students about the previous lesson and the factors one must consider to measure probability.

- Go over homework – have students share their game ideas and explanations
- Introducing probability (Slide 3)

  **Class Discussion**
  
  The probability of winning in single-bet roulette is 1 in 38. What does that mean?

  Answers vary. This question is meant to be open-ended and encourage students to talk about what probability means and explain their sense of how 1 in 38 describes probability.

- Slide 4

  **Problem**
  
  Yesterday, people said that the chances of winning the Color Pick game are 3/5.
  
  Is this right?

  Use this as an opportunity to bring up what happened yesterday and ask Orlando to reiterate his point that if the chances were closer to 50% then there would have been more winners.

**12:10pm | Make a Model (10 min)**

- Slide 5

  **Make a Model**

  - Ask students to come up with all the ways that you could have drawn a winner in the color-pick game.
  - Before you go to the tree model, give students a chance to think about this question. They might try to start making a list, but it gets hard to make a list in a systematic way.
12:20pm  Work time (20 min)
- **Slide 7.** Give students instructions for this activity:
  - Break up students into groups. Each group should get a set of color branches—one color for each of the colors used in the simulation.
  - Circulate and help students as they work.
  - Students get their leaves after they have arranged their branches and demonstrate that they understand what their leaves should look like.
  - Take a photo of each group’s branch to make a collective tree on the smart board.

12:40pm  Counting leaves (5 min)
- **Counting all leaves:** Slide 8
  - [gather photos of each group’s branch to create class tree diagram]

- **Q: Is there a way to figure this out without counting each leaf?**
  - Encourage them to use their tree diagrams to explain. The purpose of this is to have students look across all the branches and see that they are the same in structure.
  - Students should see that there are 5 ways to choose the first color. Then for each of these 5 ways, you can pair them with 4 other colors, then for each of the 20 pairs, we have 3 choices for the third color, so we multiply that by 3, or $5 \times 4 \times 3$.

12:45pm  Finding Probability (10 min)
- **Slide 9**
  - Choose/draw a winning combination.
  - Have student bring up winning leaves to the board.
  - Discuss idea that each trio can be arranged 6 different ways. In other words, each **combination** can be arranged 6 different ways.
  - Now let’s explore how many distinct combinations there are.
• You can have students identify and group together as a class by moving around the leaves and grouping them. The result should be 10 groups of 6 leaves each.
• [Alternatively, have students do the grouping first and THEN discuss what happens if when we are choosing a winner.]

**Slide 10**

- Students should see that when a trio of colors is drawn for the color pick game, there are 6 distinct “leaves” or outcomes that could be winners out of 60 total outcomes. This can be reduced to 1/10.
- Another way to see this is as 1 of 10 leaf groups. Since we agreed that order does not matter, we might consider each group of leaves as an outcome. In this case, then there is 1 group, or 1 outcome, out of 10 groups, or 10 outcomes, that would be the winner.
- From this illustration, students should realize that it is important to be consistent in thinking about how one is defining outcomes, but as long as consistency is upheld, the answer should still be the same.

**12:55 | Closing**

**Calculating Probability**

Yesterday, you estimated the probability for winning the Color Pick game.

How close were you to the actual probability you calculated today? Where would 1/10 go on our probability number line?

If time, this can launch a mini-discussion on estimating and comparing fractions.

**Exit Ticket/Homework**

**Slide 12**

**Exit Ticket/Homework**

Your friends think that the probability of winning the Color Pick game is 3/5 since they are choosing 3 colors out of 5.

Write a paragraph explaining to your friends their mistake.
Winning the Color Pick Game

When choosing 3 colors out of 5 total possible color choices, the total number of combinations is...

\[
\frac{5 \times 4 \times 3}{3 \times 2 \times 1} = \frac{60}{6} = 10
\]

There is only 1 winning combination of colors, so the probability of winning is

\[
\frac{1}{10}
\]
Chances Part 3: Winning the Sweet Million & Other Lottery Games

1. Winning the Color Pick Game
   When choosing 3 colors out of 5 total possible color choices, the total number of combinations is...

   Explain where $5 \times 4 \times 3$ comes from.
   Explain why it ends at 3 and does not keep going.
   The numerator: 60 is the total number of permutations
   The denominator: 6 is the number of ways each combination can be arranged.
   Why is 60 being divided by 6? When you divide 60 by 6, you get 10 which is all the combinations groups in.

   $\frac{5 \times 4 \times 3}{3 \times 2 \times 1} = \frac{60}{6} = 10$

   Explain where $3 \times 2 \times 1$ comes from.
   Their one set of winning colors

   There is only 1 winning combination of colors, so the probability of winning is

   What does the 1 represent?
   Only one group wins

   Why is the 10 in the denominator and the 1 in the numerator?
   Cause only one group of color wins and 10 is the total combination of colors.
**INTERVIEW SCAVENGER HUNT CARDS**

1. Based on the interviews, what are some reasons people play the lottery?  
   • Choose one or two interviews with lottery players that you think are good examples.

2. Did any lottery players win any prize money? Based on the interviews, do you think there is a pattern between how often people play the lottery and how often they win?  
   • Find one or two examples to support your answer.

3. Based on the interviews, what are some reasons people don’t play the lottery?  
   • Choose one or two interviews that you think are good examples.

4. Based on the interviews, what are the retailers’ perspectives about the lottery’s impact on the neighborhood?  
   • Choose one or two interviews with retailers that you think are good examples.
Who spends more?

<table>
<thead>
<tr>
<th>A-Ville</th>
<th>B-Towns</th>
<th>C-Burg</th>
<th>D-Corners</th>
</tr>
</thead>
</table>

3. Find neighborhoods that have a big difference in %Income spent on lottery. What might explain these differences? Why do you think people in one neighborhood might use more of their income to play the lottery than the other neighborhood?

I notice that more poor neighborhoods spend more on lottery tickets maybe because they really want to fun in that neighborhood. People in other neighborhoods may still think it's a waste so they don't spend that much.